arXiv:hep-ph/0104200v3 7 Aug 2001

Neutrino Masses, Anomalous $U(1)$ Gauge Symmetry and Doublet-Triplet Splitting

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Abstract

We propose an attractive scenario of grand unified theories in which doublet-triplet splitting is naturally realized in $SO(10)$ unification using the Dimopoulos-Wilczek mechanism. The anomalous $U(1)_A$ gauge symmetry plays an essential role in the doublet-triplet splitting mechanism. It is interesting that the anomalous $U(1)_A$ charges determine the unification scale and mass spectrum of additional particles, as well as the order of the Yukawa couplings of quarks and leptons. For the neutrino sector, bi-maximal mixing angles are naturally obtained, and proton decay via dimension 5 operators is suppressed. It is suggestive that the anomalous $U(1)_A$ gauge symmetry motivated by superstring theory effectively solves the two biggest problems in grand unified theories, the fermion mass hierarchy problem and doublet-triplet splitting problem.

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1 Introduction

The Standard Model is consistent with all present experiments. However, there are many reasons for thinking that it is not the final theory; for example, it does not explain the anomaly cancellation between quarks and leptons, the hierarchies of gauge and Yukawa couplings, charge quantization, etc. Therefore we have a strong motivation for examining the idea of grand unified theories (GUT) [1], in which the quarks and leptons are beautifully unified in several multiplets in a simple gauge group. Three gauge groups in the Standard Model are unified into a simple gauge group at a GUT scale that is considered to be just below the Planck scale. Once we accept a higher scale than the weak scale, it is one of the most promising ways to introduce supersymmetry (SUSY) around the weak scale to stabilize the weak scale. We are thus led to examine SUSY GUT [2].

However, it is not easy to obtain a realistic SUSY GUT. One of the reasons is that it is difficult to obtain a realistic fermion mass pattern in a simple way, because a unified multiplet introduces strong constraints on the Yukawa couplings of quarks and leptons. Moreover, one of the most difficult obstacles in building a realistic GUT is the "doublet-triplet (DT) splitting problem". Generally, a fine-tuning is required to obtain the light $SU(2)_L$ doublet Higgs multiplet of the weak scale while keeping the triplet Higgs sufficiently heavy to suppress the dangerous proton decay.

For the former problem, by using the information on neutrino masses obtained in recent neutrino experiments [3], there are several impressive papers [4, 5, 6, 7, 8] attempting to explain the order of the Yukawa couplings, though most of these treatments need tuning parameters to explain the large mixing angle for the atmospheric neutrino. It is natural to examine $SO(10)$ and higher gauge groups because all quarks and leptons, including the right-handed neutrino, can be unified into a single multiplet. This is important to investigate neutrino masses.

There have been several attempts to avoid the latter problem [9, 10]. One of the most promising ways to realize DT splitting in the $SO(10)$ SUSY GUT is using the Dimopoulos-Wilczek (DW) mechanism [10, 11, 12, 13]. If the adjoint field A of $SO(10)$ has a vacuum expectation value (VEV) $\langle A \rangle = i\tau_2 \times \mathrm{diag}(v, v, v, 0, 0)$, then $SO(10)$ is broken to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, and the VEV can impart masses on the triplet Higgs but not to the doublet Higgs. Unfortunately, in order to realize the DW mechanism, a rather complicated Higgs structure is required [13]. The reason is simple: The DW mechanism works essentially in a larger rank unified gauge group (like $SO(10)$ GUT) than that of the standard gauge group. For example, since the adjoint field in $SU(5)$ GUT is traceless and the rank is the same as that of the standard gauge group, it is impossible to realize the DW mechanism. On the other hand, we have to introduce VEVs of spinors C and $\bar{C}$ or other multiplets to break the remaining gauge group to the standard gauge group, because the adjoint VEV does not reduce the rank of the

$SO(10)$ gauge group. If the VEV of the spinor appears in the equation of motion that determines the adjoint VEV, the VEV of the adjoint field generally deviates from the form required for the DW mechanism. On the other hand, if the adjoint and spinor Higgs sectors are not coupled to each other in the superpotential, then pseudo Nambu-Goldstone (PNG) fields $(\mathbf{3}, \mathbf{2})_{1/6} + (\mathbf{3}, \mathbf{1})_{-2/3}$+h.c. of $SU(3)_C \times SU(2)_L \times U(1)_Y$ appear.

To avoid this problem, the adjoint field must couple to the spinor to obtain the mass of the PNG fields, retaining the DW mechanism. It is not obvious that this is possible, but in fact it is. This is possible because, for the equation of motion, the first derivative of the superpotential is important, while for the mass term, the second derivative is essential. However, usually realizing this situation requires a rather complicated Higgs sector. Recently Chacko and Mohapatra proposed a simpler model [12], in which they introduce two $\mathbf{45}$, one $\mathbf{54}$, a pair consisting of $\mathbf{16}$ and $\overline{\mathbf{16}}$, and two $\mathbf{10}$ just for the Higgs sector. Several years ago, Barr and Raby [11] examined a minimal DT splitting model that includes a single $\mathbf{45}$, two pairs of $\mathbf{16}$ and $\overline{\mathbf{16}}$ and two $\mathbf{10}$ for the Higgs sector. This simple model is very attractive. However, it requires the introduction of many singlets whose VEVs are not determined classically and must be given by hand. Moreover, in their model, dangerous terms are not forbidden by symmetry. Once the mass term A^2 and the non-renormalizable term A^4, which are essential for their model, are allowed, there is no reason to forbid higher power terms A^{2n}. With these terms, because many (infinitely many) degenerate undesired vacua appear, it is unnatural to obtain the desired DW vacuum.

In this paper, we propose a more attractive DT splitting scenario in which the GUT scale is automatically determined and the higher terms are naturally forbidden. In this scenario, the anomalous $U(1)_A$ gauge symmetry plays an essential role. Using this mechanism, a GUT with realistic Yukawa couplings can be constructed in a simple way. This model has interesting quark and lepton mass matrix structure, which predicts bi-maximal mixing in the neutrino sector.

2 Anomalous $U(1)_A$ gauge symmetry and neutrino masses

First let us recall the anomalous $U(1)_A$ gauge symmetry. It is well known that some low energy effective theories of the string theory include the anomalous $U(1)_A$ gauge symmetry, which has non-zero anomalies, such as the pure $U(1)_A^3$ anomaly, mixed anomalies with other gauge groups G_a, and a mixed gravitational anomaly [14]. These anomalies are canceled by combining the nonlinear transformation of the dilaton chiral supermultiplet D with the gauge transformation

of the $U(1)_A$ vector supermultiplet V_A as

$$V_A \rightarrow V_A + \frac{i}{2}\left(\Lambda - \Lambda^\dagger\right), \tag{1}$$

$$D \rightarrow D + \frac{i}{2}\delta_{GS}\Lambda, \tag{2}$$

where Λ is a parameter chiral superfield. This cancellation occurs because the gauge kinetic functions for V_A and the other vector supermultiplets V_a are given by

$$\mathcal{L}_{\text{gauge}} = \frac{1}{4}\int d^2\theta \left[k_A D\, W_A{}^\alpha W_{A\alpha} + k_a D\, W_a{}^\alpha W_{a\alpha} \right] + \text{h.c.}, \tag{3}$$

where $W_A{}^\alpha$ and $W_a{}^\alpha$ are the super field strengths of V_A and V_a, and k_A and k_a are Kac-Moody levels of $U(1)_A$ and G_a. The square of the gauge coupling is written in terms of the inverse of the VEV of the dilaton as $k_a \langle D \rangle = 1/g_a^2$.

The parameter δ_{GS} in Eq. (2) is related to the conditions for the anomaly cancellations,[1]

$$2\pi^2 \delta_{GS} = \frac{C_a}{k_a} = \frac{1}{3k_A}\operatorname{tr}Q_A{}^3 = \frac{1}{24}\operatorname{tr}Q_A. \tag{4}$$

The last equality is required by the cancellation of the mixed gravitational anomaly. These anomaly cancellations are understood in the context of the Green-Schwarz mechanism[15].

One of the most interesting features of the anomalous $U(1)_A$ gauge symmetry is that it induces the Fayet-Iliopoulos D-term (F-I term) radiatively[14]. Since the Kähler potential K for the dilaton D must be a function of $D + D^\dagger - \delta_{GS}V_A$ for $U(1)_A$ gauge invariance, the F-I term can be given as

$$\int d^4\theta\, K(D + D^\dagger - \delta_{GS}V_A) = \left(-\frac{\delta_{GS}K'}{2}\right)D_A + \cdots \equiv \xi^2 D_A + \cdots, \tag{5}$$

where we take the sign of Q_A so that $\xi^2 > 0$. If the Kähler potential for the dilaton is given by $K = -\ln(D + D^\dagger - \delta_{GS}V_A)$, which can be induced by a stringy calculation at tree level, ξ^2 can be approximated as

$$\xi^2 = \frac{g_s^2 \operatorname{tr}Q_A}{192\pi^2}, \tag{6}$$

where $g_s^2 = 1/\langle D \rangle$. Note that since ξ^2 is induced radiatively, the parameter ξ is expected to be smaller than the Planck scale.

When some superfields Φ_i have anomalous $U(1)_A$ charges ϕ_i, the scalar potential becomes

$$V_{\text{scalar}} = \frac{g_A^2}{2}\left(\sum_i \phi_i|\Phi_i|^2 + \xi^2\right)^2, \tag{7}$$

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[1] $C_a \equiv \operatorname{Tr}_{G_a} T(R)\,Q_A$. Here $T(R)$ is the Dynkin index of the representation R, and we use the convention in which $T(\text{fundamental rep.}) = 1/2$.

where $1/g_A^2 = k_A \langle D \rangle$. If one superfield has a negative anomalous $U(1)_A$ charge, it acquires a non-zero VEV. Below, we assume the existence of a field Φ with negative charge and normalize the anomalous $U(1)_A$ charges so that Φ has charge -1. Then the VEV of the scalar component Φ is given by

$$\langle \Phi \rangle = \xi \equiv \lambda M_P, \qquad (8)$$

which breaks the anomalous $U(1)_A$ gauge symmetry. (Here M_P is some gravity scale and is usually taken as the reduced Planck mass, $1/\sqrt{8\pi G_N}$. In the following, we use units in which $M_P = 1$.)

Next we discuss the fermion masses. In general, the Yukawa hierarchy can be explained by introducing a flavor dependent $U(1)$ symmetry [16, 17, 18, 19]. We can adopt the anomalous $U(1)_A$ gauge symmetry as this $U(1)$ symmetry. Suppose that the Standard Model matter fields Q_i, U_i^c, D_i^c, L_i, E_i^c, H_u and H_d have the anomalous $U(1)_A$ charges q_i, u_i, d_i, l_i, e_i, h_u and h_d, [2] which are taken as non-negative integers here. If the field Φ with charge -1 is a singlet under the Standard Model gauge symmetry, the superpotential can be written as

$$W \sim \Phi^{q_i + u_j + h_u} H_u Q_i U_j^c + \cdots. \qquad (9)$$

In this paper, for simplicity, we usually do not write $O(1)$ coefficients explicitly. Since the scalar component of Φ has the VEV given in (8), we obtain the hierarchical mass matrices

$$(M_u)_{ij} \sim \lambda^{q_i + u_j + h_u} \langle H_u \rangle = V_L^u \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} V_R^{u\dagger}, \qquad (10)$$

$$(M_d)_{ij} \sim \lambda^{q_i + d_j + h_d} \langle H_d \rangle = V_L^d \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} V_R^{d\dagger}, \qquad (11)$$

where the $V_{L,R}^{u,d}$ are 3×3 unitary diagonalizing matrices, and $(V_L^u)_{ij} \sim \lambda^{|q_i - q_j|}$, $(V_R^u)_{ij} \sim \lambda^{|u_i - u_j|}$, and so on. The diagonalized masses of quarks, m_f, satisfy $(m_u)_i \sim \lambda^{q_i + u_i + h_u} \langle H_u \rangle$ and $(m_d)_i \sim \lambda^{q_i + d_i + h_d} \langle H_d \rangle$. The Cabbibo-Kobayashi-Maskawa matrix [20] is given by

$$V_{\text{CKM}} = V_L^u V_L^{d\dagger} \sim \begin{pmatrix} 1 & \lambda^{|q_1 - q_2|} & \lambda^{|q_1 - q_3|} \\ \lambda^{|q_2 - q_1|} & 1 & \lambda^{|q_2 - q_3|} \\ \lambda^{|q_3 - q_1|} & \lambda^{|q_3 - q_2|} & 1 \end{pmatrix}, \qquad (12)$$

which is determined solely by the charges of the left-handed quarks, q_i. The relation $V_{12} V_{23} \sim V_{13}$ can naturally be understood with this mechanism, and if we take $q_i = (3, 2, 0)$ and $\lambda \sim 0.2$, we can obtain the measured value.

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[2] Throughout this paper we denote all the superfields with uppercase letters and their anomalous $U(1)_A$ charges with the corresponding lowercase letters.

If there are right-handed neutrinos N_i^c with $U(1)_A$ charges n_i, the Dirac and Majorana neutrino masses are also given by

$$(M_D)_{ij} \sim \lambda^{l_i+n_j+h_u} \langle H_u \rangle, \tag{13}$$

$$(M_R)_{ij} \sim M_m \lambda^{n_i+n_j}. \tag{14}$$

Through the see-saw mechanism [21], the left-handed neutrino mass matrix is given by

$$(M_\nu)_{ij} \sim \lambda^{l_i+l_j+2h_u} \frac{\langle H_u \rangle^2}{M_m}. \tag{15}$$

The mixing matrix for the lepton sector [22] is induced as for the quark sector:

$$V_{\mathrm{MNS}} = V_L^\nu V_L^{e\dagger} \sim \begin{pmatrix} 1 & \lambda^{|l_1-l_2|} & \lambda^{|l_1-l_3|} \\ \lambda^{|l_2-l_1|} & 1 & \lambda^{|l_2-l_3|} \\ \lambda^{|l_3-l_1|} & \lambda^{|l_3-l_2|} & 1 \end{pmatrix}. \tag{16}$$

This matrix is also determined only by the charges of the left-handed leptons, l_i. If we take $l_i = (2, 2, 2)$, it generally gives large mixing angles among the three generations and can give the bi-maximal mixing angles. This is called the 'anarchy solution' for large mixing angles in the neutrino sector [23].

Until this point, we have examined only terms with non-negative total anomalous $U(1)_A$ charge, but we also wish to know what happens if the total charge becomes negative. The terms with negative total anomalous $U(1)_A$ charge are forbidden by the anomalous $U(1)_A$ gauge symmetry, while the terms with positive or zero charge are allowed, because the negative charge of the singlet Φ can compensate for the positive charge, as discussed above. The vanishing of the coefficients resulting from the anomalous $U(1)_A$ gauge symmetry is called "SUSY zero" mechanism. This feature plays an essential role in our mechanism of DT splitting.

In Eq. (14), we have to introduce the Majorana mass scale M_m smaller than M_P by hand. If we simply take $M_m \sim M_P$, which is the unique scale in this model, the upper bound of the neutrino mass becomes $O(10^{-5}\mathrm{eV})$, which is much smaller than the expected values $0.04 - 0.07$ eV for the atmospheric neutrino anomaly. Here we naively expect that in the effective term (15), the factor $\lambda^{l_i+l_j+2h_u}$ cannot be larger than 1, because terms with negative total $U(1)_A$ charge $(l_i+l_j+2h_u < 0)$ must be forbidden by the SUSY zero mechanism. If we adopt the anomalous $U(1)_A$ gauge symmetry as the flavor symmetry that induces quark and lepton masses, we have to explain why the mass of right-handed neutrinos is much smaller than that expected from the anomalous $U(1)_A$ charges, or to find a way to avoid the 'SUSY zero' mechanism. One might think that introducing a singlet whose VEV gives the mass of the right-handed neutrino can allow us to avoid this problem. Unfortunately, this solution does not work well if the F-flatness condition determines the VEV. This is because, as we discuss in the next section, the

VEV of the singlet S is generally determined by the anomalous $U(1)_A$ charge s as $\langle S \rangle = \lambda^{-s}$, which does not improve the situation. Of course, we could assume the right-handed neutrino scale determined by some other conditions, for example, D-flatness conditions, SUSY breaking terms, or some dynamical mechanism. In this paper, however, we examine more appealing solutions to this problem. One of them is very simple. Note that even if we shift the anomalous $U(1)_A$ charges $(q_i, u_i, d_i, l_i, e_i, n_i, h_u, h_d)$ to $(q_i+n, u_i+n, d_i+n, l_i+n, e_i+n, n_i+n, h_u-2n, h_d-2n)$, the Dirac mass matrices of quarks and leptons remain unchanged. On the other hand the right-handed neutrino masses become smaller by a factor of λ^{2n} for positive n. Then the neutrino masses can be enhanced by a factor of λ^{-2n}. Note that even if the total charge $l_i + l_j + 2h_u$ is negative, the term in Eq. (15) is allowed. This implies that the 'SUSY zero' mechanism does not work in the effective interaction. In the effective theory, which is obtained by integrating heavy fields with positive anomalous $U(1)_A$ charges, terms with negative total charge can be induced. It is easily shown that the induced terms with negative total charge do not contribute to the F-flatness conditions if the heavy fields have vanishing VEVs. This observation is important for the DT splitting models discussed in this paper, because the SUSY zero mechanism plays an essential role to determine VEVs. Note that integrating heavy fields with masses of the Planck scale does not induce such terms, because the total $U(1)_A$ charge of the mass term is zero. This solution inevitably leads to the negative charge of the Higgs field, which is required also by the DT splitting mechanism proposed in this paper. For the other solution, which can give a smaller mass to right-handed neutrino than that expected from the anomalous $U(1)_A$ charge, it is essential that the right-handed neutrinos have the charges of a gauge interaction. We will return to this point in the next section.

3   Relation between VEVs and anomalous $U(1)_A$ charges and neutrino masses

In this section, we discuss how VEVs are determined by the anomalous $U(1)_A$ quantum numbers.

First, the VEV of a gauge invariant operator with positive anomalous $U(1)_A$ charge must vanish. Otherwise, the SUSY zero mechanism does not work, since such a VEV can compensate for the negative $U(1)_A$ charge of the term. At this stage, such an undesired vacuum is not forbidden. However, we show below that such a vacuum requires a VEV larger than the Planck scale. If the cutoff is rigid and a VEV larger than the cutoff is not allowed for some reason, then the condition for the SUSY zero mechanism is automatically satisfied.

Next we show that the VEV of a gauge invariant operator O is generally determined by its $U(1)_A$ charge o as $\langle O \rangle = \lambda^{-o}$ if the F-flatness condition deter-

mines the VEV. For simplicity, we examine this relation using singlet fields Z_i with anomalous $U(1)_A$ charge z_i. The general superpotential is written

$$W = \sum_i \lambda^{z_i} Z_i + \sum_{i,j} \lambda^{z_i+z_j} Z_i Z_j + \cdots \qquad (1)$$

$$= \sum_i \tilde{Z}_i + \sum_{i,j} \tilde{Z}_i \tilde{Z}_j + \cdots, \qquad (2)$$

where $\tilde{Z}_i \equiv \lambda^{z_i} Z_i$. The equations for the F-flatness of the Z_i fields require

$$\lambda^{z_i} \left(1 + \sum_j \tilde{Z}_j + \cdots \right) = 0, \qquad (3)$$

which generally leads to solutions $\tilde{Z}_j \sim O(1)$. Note that at least one field of a term in the superpotential must have positive or zero anomalous $U(1)_A$ charge. Otherwise we cannot write down the term satisfying $U(1)_A$ gauge invariance. As noted above, maintaining the SUSY zero feature requires that the VEV of a gauge invariant operator with positive anomalous $U(1)_A$ charge vanishes. Therefore, with this requirement, usually it is sufficient to examine the F-flatness of the gauge invariant operator with positive or zero anomalous $U(1)_A$ charges. [3] (Therefore the F-flatness condition for Φ is automatically satisfied, because Φ has negative charge -1.)

Let us return to the problem involving the mass of the neutrino, which is discussed in the previous section. From the above argument, it is shown that introducing a singlet field S with non-zero VEV and the interaction $\lambda^{s+2n^c} S N_R^c N_R^c$ cannot improve the situation because the VEV of the singlet is written $\langle S \rangle = \lambda^{-s}$ if F-flatness conditions determine the VEV. Of course it is obvious that this problem can be avoided if the VEV is determined dynamically or by some other conditions, for example, D-flatness conditions [18]. However, we now propose another simple way to avoid this problem.

Let us introduce an additional gauge freedom[4] that transforms the right-handed neutrino fields non-trivially, for example, an additional $U(1)_X$ gauge symmetry or a gauge group larger than the standard gauge group, like $SO(10)$. If the gauge variant field couples to the mass term of the right-handed neutrino and the VEV of the field breaks the additional gauge symmetry, then the coefficient can be changed. For example, if we introduce additional singlets under the standard gauge group $\Theta(1, -6)$ and $\bar{\Theta}(-1, 0)$, as well as the right-handed neutrinos $N_R^c(1, 1)$, under the gauge group $U(1)_X \times U(1)_A$, then the VEV of the gauge

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[3] Note that the F-flatness condition of gauge variant fields with negative charge can be important to determine the VEVs. This is because gauge variant fields with positive charge may have non-vanishing VEV.

[4] A global symmetry can play the same role if the Nambu-Goldstone fields are phenomenologically allowed.

invariant operator $\langle \bar{\Theta}\Theta \rangle$ is determined by the anomalous $U(1)_A$ charge -6 and is given as $\langle \bar{\Theta}\Theta \rangle = \lambda^6$. The mass term of the right-handed neutrino is obtained from the term $\lambda^2(N_R^c(1,1)\bar{\Theta}(-1,0))^2$ with the VEV $\langle \Theta(1,6) \rangle = \langle \bar{\Theta}(-1,0) \rangle \sim \lambda^3$, which is required by the D-flatness condition of $U(1)_X$. This mass term is of order λ^8, which is much smaller than the naively expected value λ^2. The fact that the additional gauge freedom is required to obtain the correct size of the mass of the right-handed neutrino implies that the GUT, if it exists, must have a rank greater than 4. The $SO(10)$ gauge group is one of the most promising possibility, because it also unifies one generation of quarks and leptons, including the right-handed neutrino, in a single multiplet (spinor) Ψ. Actually if we adopt the $SO(10)$ gauge group, which is broken by the VEV of the spinor $\langle C \rangle = \langle \bar{C} \rangle \sim \lambda^{-(c+\bar{c})/2}$, the mass term of the right-handed neutrino is given from the term $\lambda^{2(\psi+\bar{c})}(\Psi\bar{C})^2$. The mass $\lambda^{2\psi+\bar{c}-c}$ can be smaller than the naively expected value $\lambda^{2\psi}$. The model proposed by Bando and Kugo [6] has such a structure in E_6 unification.

Such a solution, employing a larger unification group, is attractive, but GUT generally suffers from the DT splitting problem. In the next section we show that the DT splitting is naturally realized in $SO(10)$ unification using the anomalous $U(1)_A$ gauge symmetry.

4 Doublet-triplet splitting with anomalous $U(1)_A$ gauge symmetry

In the previous section, we emphasized that introducing the $SO(10)$ grand unified group or a larger group can naturally explain the mass scale of the right-handed neutrino. However, if we proceed to the unified theory with a simple group, we have to solve the doublet-triplet splitting problem. In this section, we propose an $SO(10)$ unified model that naturally realizes the doublet-triplet splitting.

The content of the Higgs sector with $SO(10)\times U(1)_A$ gauge symmetry is given in Table I, where the symbols $\pm$ denote a parity quantum numbers.

Table I. The lowercase letters represent the anomalous $U(1)_A$ charges.

$$
\begin{aligned}
\mathbf{45} &: \quad A(a=-2,-), \quad A'(a'=6,-) \\
\mathbf{16} &: \quad C(c=-1,+), \quad C'(c'=8,-) \\
\overline{\mathbf{16}} &: \quad \bar{C}(\bar{c}=-5,+), \quad \bar{C}'(\bar{c}'=4,-) \\
\mathbf{10} &: \quad H(h=-2,+), \quad H'(h'=4,-) \\
\mathbf{1} &: \quad Z(z=-3,-), \quad \bar{Z}(\bar{z}=-3,-), \quad S(s=5,+)
\end{aligned}
$$

Here we have listed typical values of the anomalous $U(1)_A$ charges. Among these fields, A, C, $\bar{C}$, Z and $\bar{Z}$ are expected to obtain non-vanishing VEVs around the

GUT scale. Here, for simplicity we assume that the fields with positive $U(1)_A$ charges have vanishing VEVs, although we can give a more rigorous argument for this.

Since the fields with non-vanishing VEVs have negative charges, only the F-flatness conditions of fields with positive charge must be counted for determination of their VEVs. (Generally c or $\bar{c}$ can be positive, although we are now considering $c = -1$ and $\bar{c} = -5$, because it is sufficient for maintaining SUSY zero mechanism that the charge $c + \bar{c}$ of the gauge invariant operator $\bar{C}C$ become non-positive. The following argument does not change significantly if c or $\bar{c}$ is positive.) Moreover, we have only to take account of the terms in the superpotential which contain only one field with positive charge. This is because the terms with more positive charge fields do not contribute to the F-flatness conditions, since the positive fields are assumed to have zero VEV. Therefore, in general, the superpotential required by determination of the VEVs can be written as

$$W = W_{H'} + W_{A'} + W_S + W_{C'} + W_{\bar{C}'}. \tag{1}$$

Here W_X denotes the terms linear in the X field, which has positive anomalous $U(1)_A$ charge. Note, however, that terms including two fields with positive charge like $\lambda^{2h'}H'H'$ give contributions to the mass terms but not to the VEVs. $W_{A'}$ can realize the DW form for the VEV of A, $\langle A \rangle = i\tau_2 \times \mathrm{diag}(v, v, v, 0, 0)$, which is proportional to the generator $B - L$. Such a VEV of A gives a super heavy mass to the color triplets in H and H' through the $W_{H'} = HAH'$ term, while keeping the weak doublets massless. This implies that the F-flatness condition of H' causes a vanishing VEV of the colored Higgs in H, but not the VEV of the doublet Higgs in H. The mass term $\lambda^{2h'}(H')^2$ imparts a mass $\sim \lambda^{2h'}$ on the extra doublet in H'. Therefore it is realized that only one pair of doublet Higgs in H becomes massless.

We now discuss the determination of the VEVs. For determination of the VEVs, it is sufficient to take account of the superpotential terms, which include only fields with non-zero VEVs, except one field with vanishing VEV. If $-3a \leq a' < -5a$, the superpotential $W_{A'}$ is in general written as

$$W_{A'} = \lambda^{a'+a}\alpha A'A + \lambda^{a'+3a}(\beta(A'A)_{\mathbf{1}}(A^2)_{\mathbf{1}} + \gamma(A'A)_{\mathbf{54}}(A^2)_{\mathbf{54}}), \tag{2}$$

where the suffixes $\mathbf{1}$ and $\mathbf{54}$ indicate the representation of the composite operators under the $SO(10)$ gauge symmetry, and α, β and γ are parameters of order 1. Here we assume $a + a' + c + \bar{c} < 0$ to forbid the term $\bar{C}A'AC$, which destabilizes the DW form of the VEV $\langle A \rangle$. If we take $\langle A \rangle = i\tau_2 \times \mathrm{diag}(x_1, x_2, x_3, x_4, x_5)$, the F-flatness of the A' field requires $x_i(\alpha\lambda^{-2a} + 2(\beta - \gamma)(\sum_j x_j^2) + \gamma x_i^2) = 0$, which gives only two solutions $x_i^2 = 0$, $\frac{\alpha}{(2N-1)\gamma-2N\beta}\lambda^{-2a}$. Here $N = 1-5$ is the number of $x_i \neq 0$ solutions. The DW form is obtained when $N = 3$. Note that the higher terms $A'A^{2L+1}$ ($L > 1$) are forbidden by the SUSY zero mechanism. If they are allowed, the number of possible VEVs other than the DW form becomes larger,

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and thus it becomes less natural to obtain the DW form. This is a critical point of this mechanism, and the anomalous $U(1)_A$ gauge symmetry plays an essential role to forbid the undesired terms. It is also interesting that the scale of the VEV is automatically determined by the anomalous $U(1)_A$ charge of A, as noted in the previous section.

Next we discuss the F-flatness condition of S, which determines the scale of the VEV $\langle \bar{C}C \rangle$. W_S, which is linear in the S field, is given by

$$W_S = \lambda^{s+c+\bar{c}} S \left((\bar{C}C) + \lambda^{-(c+\bar{c})} + \sum_k \lambda^{-(c+\bar{c})+2ka} A^{2k} \right) \tag{3}$$

if $s \geq -(c + \bar{c})$. Then the F-flatness condition of S implies $\langle \bar{C}C \rangle \sim \lambda^{-(c+\bar{c})}$, and the D-flatness condition requires $|\langle C \rangle| = |\langle \bar{C} \rangle| \sim \lambda^{-(c+\bar{c})/2}$. The scale of the VEV is determined only by the charges of C and $\bar{C}$ again. If we take $c + \bar{c} = -6$, then we obtain the VEVs of the fields $\bar{C}$ and $\bar{C}$ as λ^3, which differ from the expected values λ^{-c} and $\lambda^{-\bar{c}}$ if $c \neq \bar{c}$. Note that a composite operator with positive anomalous $U(1)_A$ charge larger than $-(c+\bar{c})-1$ may play the same role as the singlet S if such a composite operator exists. (In the above example, there is no such composite operator.)

Finally, we discuss the F-flatness of C' and $\bar{C}'$, which realizes the alignment of the VEVs $\langle C \rangle$ and $\langle \bar{C} \rangle$ and imparts masses on the PNG fields. This simple mechanism was proposed by Barr and Raby [11]. We can easily assign anomalous $U(1)_A$ charges which allow the following superpotential:

$$W_{C'} = \bar{C}(\lambda^{\bar{c}'+c+a} A + \lambda^{\bar{c}'+c+\bar{z}} \bar{Z}) C', \tag{4}$$

$$W_{\bar{C}'} = \bar{C}'(\lambda^{\bar{c}'+c+a} A + \lambda^{\bar{c}'+c+z} Z) C. \tag{5}$$

The F-flatness conditions $F_{C'} = F_{\bar{C}'} = 0$ give $(\lambda^{a-z} A + Z)C = \bar{C}(\lambda^{a-\bar{z}} A + \bar{Z}) = 0$. Recall that the VEV of A is proportional to the $B - L$ generator Q_{B-L} as $\langle A \rangle = \frac{3}{2} v Q_{B-L}$. Also C, $\mathbf{16}$, is decomposed into $(\mathbf{3}, \mathbf{2}, \mathbf{1})_{1/3}$, $(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{2})_{-1/3}$, $(\mathbf{1}, \mathbf{2}, \mathbf{1})_{-1}$ and $(\mathbf{1}, \mathbf{1}, \mathbf{2})_1$ under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Since $\langle \bar{C}C \rangle \neq 0$, not all components in the spinor C vanish. Then Z is fixed to be $Z \sim -\frac{3}{2} \lambda v Q_{B-L}^0$, where Q_{B-L}^0 is the $B - L$ charge of the component field in C, which has non-vanishing VEV. It is interesting that no other component fields can have non-vanishing VEVs because of the F-flatness conditions. If the $(\mathbf{1}, \mathbf{1}, \mathbf{2})_1$ field obtains a non-zero VEV (therefore, $\langle Z \rangle \sim -\frac{3}{2} \lambda v$), then the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is broken to the standard gauge group. Once the direction of the VEV $\langle C \rangle$ is determined, the VEV $\langle \bar{C} \rangle$ must have the same direction because of the D-flatness condition. Therefore, $\langle \bar{Z} \rangle \sim -\frac{3}{2} \lambda v$. Thus, all VEVs have now been fixed.

Next we examine the mass spectrum. Since for the mass terms, we must take account of not only the above terms but also the terms that contain two fields with vanishing VEVs.

Considering the additional mass term $\lambda^{2h'}H'H'$, we write the mass matrix of the Higgs fields H and H', which are decomposed from $\mathbf{5}$ and $\bar{\mathbf{5}}$ of $SU(5)$, as

$$(\mathbf{5}_H, \mathbf{5}_{H'}) \begin{pmatrix} 0 & \lambda^{h+h'+a}\langle A \rangle \\ \lambda^{h+h'+a}\langle A \rangle & \lambda^{2h'} \end{pmatrix} \begin{pmatrix} \bar{\mathbf{5}}_H \\ \bar{\mathbf{5}}_{H'} \end{pmatrix}. \tag{6}$$

The colored Higgs obtain their masses of order $\lambda^{h+h'+a}\langle A \rangle \sim \lambda^{h+h'}$. Since in general $\lambda^{h+h'} > \lambda^{2h'}$, the proton decay is naturally suppressed. The effective colored Higgs mass is estimated as $(\lambda^{h+h'})^2/\lambda^{2h'} = \lambda^{2h}$, which is larger than the Planck scale, because $h < 0$. One pair of the doublet Higgs is massless, while another pair of doublet Higgs acquires a mass of order $\lambda^{2h'}$, which is $\sim \lambda^8 \sim 5 \times 10^{12}$ GeV in the typical $U(1)_A$ assignment in Table I. The DW mechanism works well, although we have to examine the effect of the rather light additional Higgs.

Next we examine the mass matrices for the representations $I = Q, U^c$ and E^c, which are contained in the $\mathbf{10}$ of $SU(5)$. Like the superpotential previously discussed, the additional terms $\lambda^{2a'}A'A'$, $\lambda^{c'+\bar{c}'}\bar{C}'C'$, $\lambda^{c'+a'+\bar{c}}\bar{C}A'C'$ and $\lambda^{\bar{c}'+a'+c}\bar{C}'A'C$ must be taken into account. The mass matrices are written as 4×4 matrices,

$$\left(\bar{I}_A, \bar{I}_{A'}, \bar{I}_{\bar{C}}, \bar{I}_{\bar{C}'}\right) \begin{pmatrix} 0 & \lambda^{a'+a}\alpha_I & 0 & \frac{\lambda^{\bar{c}+c'+a}}{\sqrt{2}}\langle \bar{C} \rangle \\ \lambda^{a+a'}\alpha_I & \lambda^{2a'} & 0 & \frac{\lambda^{\bar{c}+c'+a'}}{\sqrt{2}}\langle \bar{C} \rangle \\ 0 & 0 & 0 & \lambda^{\bar{c}+c'+a}\beta_I v \\ \frac{\lambda^{c+\bar{c}'+a}}{\sqrt{2}}\langle C \rangle & \frac{\lambda^{c+\bar{c}'+a'}}{\sqrt{2}}\langle C \rangle & \lambda^{c+\bar{c}'+a}\beta_I v & \lambda^{c'+\bar{c}'} \end{pmatrix} \begin{pmatrix} I_A \\ I_{A'} \\ I_C \\ I_{C'} \end{pmatrix}, \tag{7}$$

where α_I vanishes for $I = Q$ and U^c because these are Nambu-Goldstone modes, but $\alpha_{E^c} \neq 0$. On the other hand, $\beta_I = \frac{3}{2}((B-L)_I - 1)$; that is, $\beta_Q = -1$, $\beta_{U^c} = -2$ and $\beta_{E^c} = 0$. Thus for each I, the 4×4 matrix has one vanishing eigenvalue, which corresponds to the Nambu-Goldstone mode eaten by the Higgs mechanism. The mass spectrum of the remaining three modes is $(\lambda^{c+\bar{c}'+a}v, \lambda^{c'+\bar{c}+a}v, \lambda^{2a'})$ for the color-triplet modes Q and U^c, and $(\lambda^{a+a'}, \lambda^{a+a'}, \lambda^{c'+\bar{c}'})$ or $(\lambda^{c+\bar{c}'+a}\langle C \rangle, \lambda^{c'+\bar{c}+a}\langle \bar{C} \rangle, \lambda^{2a'})$ for the color-singlet modes E^c. (These are dependent on the anomalous $U(1)_A$ charges.) If we use typical anomalous $U(1)_A$ charges, as listed in the previous table, then the light modes are Q, U^c, E^c and their hermitian conjugate fields, which are contained in a pair of $\mathbf{10}$ and $\overline{\mathbf{10}}$ of $SU(5)$, with a mass of order $\lambda^{12} \sim 10^{10}$ GeV. Though in principle the mass of the color-triplet fields and that of the color-singlet field are determined independently, it is interesting that all the fields in a single multiplet $\mathbf{10}$ of $SU(5)$ become light together. This fact makes us expect that the success of the gauge coupling unification may not be drastically changed with these light modes.

If we simply omit the rows and columns of A and A' in Eq. (7), then we obtain 2×2 mass matrices, which are for the representations D^c and L and their

conjugates. Since $\beta_{D^c} = -2$ and $\beta_L = -3$, the color triplets acquire masses $2\lambda^{\bar{c}+c'}$ and $2\lambda^{c+\bar{c}'}$, while the weak doublets acquire masses $3\lambda^{\bar{c}+c'}$ and $3\lambda^{c+\bar{c}'}$.

The adjoint fields A and A' contain two $(\mathbf{8}, \mathbf{1})_0$ and two $(\mathbf{1}, \mathbf{3})_0$ of the standard gauge group, which acquire mass $\lambda^{a'+a}$. Moreover, they contain two pairs of $(\mathbf{3}, \mathbf{2})_{-5/6}$+h.c. One of these is eaten by Higgs mechanism, but another pair has a rather light mass of $\lambda^{2a'}$, which may destroy the coincidence of the running gauge couplings.

Once we determine the anomalous $U(1)_A$ charges, the mass spectrum of all fields is determined, and hence we can calculate the Weinberg angle. However, since the estimation is strongly dependent on the assignment of the anomalous $U(1)_A$ charges, as shown in the above argument, and on the details of the DT splitting sector and the matter sector, we do not discuss it further here.

There are several terms which must be forbidden for the stability of the DW mechanism. For example, H^2, HZH' and $H\bar{Z}H'$ induce a large mass of the doublet Higgs, and the term $\bar{C}A'AC$ would destabilize the DW form of $\langle A \rangle$. We can easily forbid these terms using the SUSY zero mechanism. For example, if we choose $h < 0$, then H^2 is forbidden, and if we choose $\bar{c} + c + a + a' < 0$, then $\bar{C}A'AC$ is forbidden. (It is interesting that the negative $U(1)_A$ charge h, which is required for the DT splitting, enhances the left-handed neutrino masses, as discussed in section 2.) Once these dangerous terms are forbidden by the SUSY zero mechanism, higher-dimensional terms which also become dangerous; for example, $\bar{C}A'A^3C$ and $\bar{C}A'C\bar{C}AC$ are automatically forbidden, since only gauge invariant operators with negative charge can have non-vanishing VEVs. This is also an attractive point of our scenario. Actually, the symmetry discussed in Ref.[7] does not forbid the dangerous term $(\bar{C}AC)^2$, which destabilizes the DW form of $\langle A \rangle$.

In this section, we have proposed an natural DT splitting mechanism in which the anomalous $U(1)_A$ gauge symmetry play a critical role, and the VEVs and mass spectrum are automatically determined by the anomalous $U(1)_A$ charges. In the next section, we examine the simplest model with this DT splitting mechanism, which gives realistic mass matrices of quarks and leptons.

5 The simplest model

In this section, we examine the simplest model to demonstrate how to determine everything from the anomalous $U(1)_A$ charges.

In addition to the Higgs sector in Table.I, we introduce only three $\mathbf{16}$ representations Ψ_i with anomalous $U(1)_A$ charges ($\psi_1 = n + 3, \psi_2 = n + 2, \psi_3 = n$) and one $\mathbf{10}$ field T with charge t as the matter content. These matter fields are assigned odd R-parity, while those of the Higgs sector are assigned even R-parity. Such an assignment of R-parity guarantees that the argument regarding VEVs in the previous section does not change if these matter fields have vanishing VEVs.

We can give an argument to determine the allowed region of the anomalous $U(1)_A$ charges to obtain desired terms while forbidding dangerous terms. Though this is a straightforward argument, we do not give it here. Instead, we give a set of anomalous $U(1)_A$ charges with which all conditions are satisfied and a novel neutrino mass matrix is obtained: $n = 3, t = 4, h = -6, h' = 8, c = -4, \bar{c} = -1, c' = 4, \bar{c}' = 7, s = 5$. Then the mass term of $\mathbf{5}$ and $\bar{\mathbf{5}}$ of $SU(5)$ is written as

$$\mathbf{5}_T (\lambda^6 \langle C \rangle, \lambda^5 \langle C \rangle, \lambda^3 \langle C \rangle, \lambda^8) \begin{pmatrix} \bar{\mathbf{5}}_{\Psi 1} \\ \bar{\mathbf{5}}_{\Psi 2} \\ \bar{\mathbf{5}}_{\Psi 3} \\ \bar{\mathbf{5}}_{\mathbf{T}} \end{pmatrix}. \tag{1}$$

Since $\langle \bar{C} \rangle = \langle C \rangle \sim \lambda^{5/2}$, because $c + \bar{c} = -5$, the massive mode $\bar{\mathbf{5}}_M$, the partner of 5_T, is given by

$$\bar{\mathbf{5}}_M \sim \bar{\mathbf{5}}_{\Psi 3} + \lambda^{5/2} \bar{\mathbf{5}}_T. \tag{2}$$

Therefore the three massless modes $(\bar{\mathbf{5}}_1, \bar{\mathbf{5}}_2, \bar{\mathbf{5}}_3)$ are written $(\bar{\mathbf{5}}_{\Psi 1}, \bar{\mathbf{5}}_T + \lambda^{\frac{5}{2}} \bar{\mathbf{5}}_{\Psi 3}, \bar{\mathbf{5}}_{\Psi 2})$. The Dirac mass matrices for quarks and leptons can be obtained from the interaction

$$\lambda^{\psi_i + \psi_j + h} \Psi_i \Psi_j H. \tag{3}$$

The mass matrices for the up quark sector and the down quark sector are

$$M_u = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle, \quad M_d = \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^{7/2} & \lambda^3 \\ \lambda^3 & \lambda^{5/2} & \lambda^2 \\ \lambda^1 & \lambda^{1/2} & 1 \end{pmatrix} \langle H_d \rangle. \tag{4}$$

Note that the Yukawa couplings for $\bar{\mathbf{5}}_2 \sim \bar{\mathbf{5}}_T + \lambda^{5/2} \bar{\mathbf{5}}_{\Psi 3}$ are obtained only through the Yukawa couplings for the component $\bar{\mathbf{5}}_{\Psi 3}$, because we have no Yukawa couplings for T. We can estimate the CKM matrix from these quark matrices as

$$U_{CKM} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \tag{5}$$

which is consistent with the experimental value if we choose $\lambda \sim 0.2$. Since the ratio of the Yukawa couplings of top and bottom quarks is λ^2, a small value of $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle \sim O(1)$ is predicted by these mass matrices. The Yukawa matrix for the charged lepton sector is the same as the transpose of M_d at this stage, except for an overall factor η induced by the renormalization group effect. The mass matrix for the Dirac mass of neutrinos is given by

$$M_D = \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda \\ \lambda^{7/2} & \lambda^{5/2} & \lambda^{1/2} \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle \, \eta. \tag{6}$$

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The right-handed neutrino masses come from the interaction

$$\lambda^{\psi_i+\psi_j+2\bar{c}}\Psi_i\Psi_j\bar{C}\bar{C} \tag{7}$$

as

$$M_R = \lambda^{\psi_i+\psi_j+2\bar{c}}\left\langle\bar{C}\right\rangle^2 = \lambda^9\begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \tag{8}$$

Therefore we can estimate the neutrino mass matrix:

$$M_\nu = M_D M_R^{-1} M_D^T = \lambda^{-5}\begin{pmatrix} \lambda^2 & \lambda^{3/2} & \lambda \\ \lambda^{3/2} & \lambda & \lambda^{1/2} \\ \lambda & \lambda^{1/2} & 1 \end{pmatrix}\left\langle H_u\right\rangle^2\eta^2. \tag{9}$$

Note that the overall factor λ^{-5} has negative power, which can be induced by the effects discussed in sections 2 and 3. From these mass matrices in the lepton sector the MNS matrix is obtained as

$$U_{MNS} = \begin{pmatrix} 1 & \lambda^{1/2} & \lambda \\ \lambda^{1/2} & 1 & \lambda^{1/2} \\ \lambda & \lambda^{1/2} & 1 \end{pmatrix}. \tag{10}$$

This gives bi-maximal mixing angles for the neutrino sector, because $\lambda^{1/2} \sim 0.5$.[5] We then obtain the prediction $m_{\nu_\mu}/m_{\nu_\tau} \sim \lambda$, which is consistent with the experimental data: $1.6\times10^{-3}(\text{eV})^2 \leq \Delta m_{\text{atm}}^2 \leq 4\times10^{-3}(\text{eV})^2$ and $2\times10^{-5}(\text{eV})^2 \leq \Delta m_{\text{solar}}^2 \leq 1\times10^{-4}(\text{eV})^2$. The relation $V_{e3} \sim \lambda$ is also an interesting prediction from this matrix, though CHOOZ gives a restrictive upper limit $V_{e3} \leq 0.15$ [25]. The neutrino mass is given by $m_{\nu_\tau} \sim \lambda^{-5}\left\langle H_u\right\rangle^2\eta^2/M_P \sim m_{\nu_\mu}/\lambda \sim m_{\nu_e}/\lambda^2$. If we take $\left\langle H_u\right\rangle\eta = 100\,\text{GeV}$, $M_P \sim 10^{18}\,\text{GeV}$ and $\lambda = 0.2$, then we get $m_{\nu_\tau} \sim 3\times10^{-2}$ eV, $m_{\nu_\mu} \sim 6\times10^{-3}$ eV and $m_{\nu_e} \sim 1\times10^{-3}$ eV. It is surprising that such a rough approximation gives values in good agreement with the experimental values from the atmospheric neutrino and large mixing angle (LMA) MSW solution of the solar neutrino problem [26]. This LMA solution for the solar neutrino problem gives the best fit to the present experimental data [27].

In addition to Eq. (3), the interactions

$$\lambda^{\psi_i+\psi_j+2a+h}\Psi_i A^2 \Psi_j H \tag{11}$$

also contribute to the Yukawa couplings. Here A is squared because it has odd parity. Since A is proportional to the generator of $B-L$, the contribution to the lepton Yukawa coupling is nine times larger than that to quark Yukawa coupling, which can change the unrealistic prediction $m_\mu = m_s$ at the GUT scale. Since

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[5] After submitting this paper, we noticed the reference [24] in which this neutrino mass structure has been discussed with the semi-simple unified group $SU(6) \times SU(2)_R$.

the prediction $m_s/m_b \sim \lambda^{5/2}$ at the GUT scale is consistent with experiment, the enhancement factor $2 \sim 3$ of m_μ can improve the situation. Note that the additional terms contribute mainly in the lepton sector. If we set $a = -2$, the additional matrices are

$$\frac{\Delta M_u}{\langle H_u \rangle} = \frac{v^2}{4} \begin{pmatrix} \lambda^2 & \lambda & 0 \\ \lambda & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \frac{\Delta M_d}{\langle H_d \rangle} = \frac{v^2}{4} \begin{pmatrix} \lambda^2 & 0 & \lambda \\ \lambda & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \tag{12}$$

$$\frac{\Delta M_e}{\langle H_d \rangle} = \frac{9v^2}{4} \begin{pmatrix} \lambda^2 & \lambda & 0 \\ 0 & 0 & 0 \\ \lambda & 1 & 0 \end{pmatrix}. \tag{13}$$

It is interesting that this modification essentially changes the eigenvalues of only the first and second generation. Therefore it is natural to expect that a realistic mass pattern can be obtained by this modification. This is one of the largest motivations to choose $a = -2$. Note that this charge assignment also determines the scale $\langle A \rangle \sim \lambda^2$. It is suggestive that the fact that the $SO(10)$ breaking scale is slightly smaller than the Planck scale is correlated with the discrepancy between the naive prediction of the ratio m_μ/m_s from the unification and the experimental value. It is also interesting that the SUSY zero mechanism plays an essential role again. When $z, \bar{z} \geq -4$, the terms $\lambda^{\psi_i+\psi_j+a+z+h} Z\Psi_i A\Psi_j H + \lambda^{\psi_i+\psi_j+2z+h} Z^2 \Psi_i \Psi_j H$ also contribute to the fermion mass matrices, though only to the first generation.

Proton decay mediated by the colored Higgs is strongly suppressed in this model. As mentioned in the previous section, the effective mass of the colored Higgs is of order $\lambda^{2h} \sim \lambda^{-12}$, which is much larger than the Planck scale. Proton decay is also induced by the non-renormalizable term

$$\lambda^{\psi_i+\psi_j+\psi_k+\psi_l} \Psi_i \Psi_j \Psi_k \Psi_l, \tag{14}$$

which is also strongly suppressed.

Unfortunately, in this model we obtain an extra light doublet Higgs with mass of order $\lambda^{2h'} \sim \lambda^{16}$ and extra fields $(\mathbf{3}, \mathbf{2})_{-5/6} + \text{h.c.}$ with mass of order $\lambda^{2a'} \sim \lambda^{12}$, which may destroy gauge coupling unification. Actually, a rough approximation shows that the meeting point of the gauge couplings of $SU(3)_C$ and $SU(2)_L$ is too low to maintain the proton stability for any $U(1)_A$ charge assignment. However, since the mass spectrum is strongly dependent on the details of DT splitting models and the matter sector, we expect that for a certain charge assignment of a certain DT splitting model and the matter sector, the coupling unification is recovered. In other words, the requirement of coupling unification represents a strong constraint on these models.

6 Discussion

In this paper, we have examined a DT splitting model and emphasized that the anomalous $U(1)_A$ gauge symmetry plays an essential role to realize DT splitting

by the DW mechanism. This statement is generally true. Actually, we can make various types of DT splitting models in which the anomalous $U(1)_A$ gauge symmetry plays the role discussed in this paper. For example, if we exchange the parity between C' and $\bar{C}$, the term $CC'H$ is allowed. After obtaining the VEV of the C field, the massless doublet Higgs becomes a linear combination of $\bar{\mathbf{5}}_H$ and $\bar{\mathbf{5}}_{C'}$ of $SU(5)$. This may give richer structure to the quark and lepton matrices, though a dangerous term $CA'\bar{C}$ must be forbidden by the SUSY zero mechanism. We can introduce an additional Higgs pair, $F : \mathbf{16}$ and $\bar{F} : \overline{\mathbf{16}}$, to obtain a massless doublet Higgs that is linear combination of $\bar{\mathbf{5}}_H$ and $\bar{\mathbf{5}}_F$. Yet another way to modify the DT splitting model is to introduce an additional adjoint field $\mathbf{45}$ $A_+(a_+, +)$ with $a_+ < 0$. Then the mass spectrum of light modes is quite different from that of the model studied in this paper, because of the term $A'A_+A$. Moreover, we have assumed that the anomalous $U(1)_A$ charges take integer values, but in principle, they can take rational values as in Ref.[8]. We have not carefully examined all these modified DT splitting models. We will examine various possibilities in the future. The condition for gauge coupling unification must be a useful guide to select these models.

In principle, we may adopt an anomaly-free $U(1)_A$ gauge symmetry and the F-I D-term instead of the anomalous $U(1)_A$ gauge symmetry, though it seems to be difficult to find a consistent $U(1)_A$ charge assignment. Moreover, since we have no reason to choose the scale of the F-I D-term to be less than the Planck scale, we think it more natural to adopt the anomalous $U(1)_A$ gauge symmetry.

Finally, we discuss SUSY breaking. Since the anomalous $U(1)_A$ charges should depend on the flavor to produce a hierarchy of Yukawa couplings, generally non-degenerate scalar fermion masses are induced through the anomalous $U(1)_A$ D-term. The large SUSY breaking scale allows us avoid the flavor changing neutral current (FCNC) problem [28, 29], but in the present scenario it does not work, because the anomalous $U(1)_A$ charge of the Higgs H must be negative to forbid the Higgs mass term at tree level. Therefore the anomalous $U(1)_A$ D-term contribution, which is dependent on the flavor, must be dominated by other flavor-independent contribution to the sfermion mass terms. In principle, it is possible, for example, that the F-term of the dilation field dominates the dangerous D-term contribution. In fact, Arkani-Hamed, Dine and Martin [30] pointed out that the F-term contribution of the dilaton field can be larger than the anomalous $U(1)_A$ D-term contribution, depending on how the dilaton is stabilized, even in the case that the anomalous $U(1)_A$ gauge symmetry triggers SUSY breaking[31, 32]. [6] It is interesting that the lepton flavor violation process can be seen in this scenario [33]. Since the FCNC process introduces severe constraints on the ratio of the D-term contribution and the flavor independent contribution, it is valuable to examine the condition for which the constraints become weaker. If the first gen-

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[6] In our case, it is difficult for the anomalous $U(1)_A$ gauge symmetry to trigger SUSY breaking, because we have many fields with negative charge in addition to the field Φ.

eration of $\bar{\mathbf{5}}$ of $SU(5)$ has the same charge as the second generation, the constraint becomes much weaker [33]. The condition for the model discussed in this paper is $\psi_1 = t$. If we assign the anomalous $U(1)_A$ charge as $n = 5, t = 8, a = -2, a' = 6, h = -10, h' = 12, c = -2, \bar{c} = -3, c' = 6, \bar{c}' = 5, z = \bar{z} = -3, s = 5$, the above situation is realized, although the mass of an additional pair of Higgs becomes λ^{24}. Even if the above effect is negligible, the lepton flavor violation process may be seen through the renormalization effect of the left-handed slepton masses [34].

In subsequent papers [35], it is shown that the DT splitting mechanism can be non-trivially incorporated into E_6 unification. It is interesting that the mass matrices with bi-maximal mixing discussed in this paper appear again in the E_6 unified model. Moreover, the above condition $\psi_1 = t$, which makes the constraints from the FCNC process weaker, is automatically satisfied.

7   Conclusion

In this paper, we have pointed out that, in order to realize the correct size of the neutrino mass for the atmospheric neutrino anomaly with the anomalous $U(1)_A$ gauge symmetry, it is natural to introduce a Higgs field with negative $U(1)_A$ charge and a gauge group under which the right-handed neutrino transforms non-trivially, for example, $SO(10)$, E_6, or extra $U(1)$. Next we proposed an $SO(10)$ unified model in which DT splitting is naturally realized by the DW mechanism. The anomalous $U(1)_A$ gauge symmetry plays an essential role in the DT splitting. Using this mechanism, we examined the simplest model in which realistic mass matrices of quarks and leptons, including the neutrino, can be determined by the anomalous $U(1)_A$ charges. This model predicts bi-maximal mixing angles in the neutrino sector, a small value of $\tan\beta$, and the relation $V_{e3} \sim \lambda$. Proton stability is naturally realized. It is interesting that once we fix the anomalous $U(1)_A$ charges for all fields, the order of each parameter and scale is determined, except that of the SUSY breaking.

It is very suggestive that the anomalous $U(1)_A$ gauge symmetry motivated by superstring theory plays a critical role in solving the two biggest problems in GUT, the fermion mass hierarchy problem and the doublet-triplet splitting problem. This may be the first evidence for the validity of string theory from the phenomenological point of view.

8   Acknowledgements

we obtained an opportunity to reconsider the mass of neutrino with anomalous $U(1)_A$ gauge symmetry.

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